Worksheet 3

Name: _____ Score: _____

1. Let $f : \mathbb{P}_2 \to \mathbb{R}^2$ be the linear map from polynomials of degree less than or equal to 2 to the plane defined by

$$f(p(t)) = \begin{bmatrix} p'(1) - p''(2) \\ \int_0^1 p(t)dt \end{bmatrix}.$$

Here $p'(t), p''(t)$ are the first and second derivative and $\int_0^1 -dt$ is the definite integral.

Write f as a matrix with respect to the standard basis on $\mathbb{R}^2$ and the basis $\{1, t, t^2\}$ for $\mathbb{P}_2$.

$$[f] = \begin{bmatrix} \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_ \\ \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_ \end{bmatrix}$$

2. Write the change-of-basis matrix $\underset{\mathscr{C} \leftarrow \mathscr{B}}{P}$ for each pair of bases for $\mathbb{R}^2$:

(a)

$$\mathscr{B} = \left\{ \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 5 \\ -3 \end{bmatrix} \right\}$$

$$\mathscr{C} = \left\{ \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \end{bmatrix} \right\}$$

$$\underset{\mathscr{C} \leftarrow \mathscr{B}}{P} = \begin{bmatrix} \_\_\_\_\_ & \_\_\_\_\_ \\ \_\_\_\_\_ & \_\_\_\_\_ \end{bmatrix}$$

(b)

$$\mathscr{B} = \left\{ \begin{bmatrix} 4 \\ -3 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \end{bmatrix} \right\}$$

$$\mathscr{C} = \left\{ \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \end{bmatrix} \right\}$$

$$\underset{\mathscr{C} \leftarrow \mathscr{B}}{P} = \begin{bmatrix} \_\_\_\_\_ & \_\_\_\_\_ \\ \_\_\_\_\_ & \_\_\_\_\_ \end{bmatrix}$$